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NUMBER REPRESENTATION IN THE BRAIN USING MEG

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Abstract. Most people use natural numbers daily for counting, estimating quantities, telling time, etc. Numbers are most commonly represented using words (e.g., three), Arabic digits (e.g., 3), Roman numerals (e.g., III), or repeated symbols (e.g., ♡♡♡). However, numeration in the brain is still not well understood. There have been many attempts to use functional neuroimaging to identify brain regions that support simple numerical processes. However, these studies are not refined enough to distinguish between different numerical processes, and our understanding of brain activity during numerical processes is continuing to evolve. In this paper, we proved that there is an abstract concept of a number in the brain and we observed different properties of the MEG's (magnetoencephalography) representation of brain activity while encountering stimulation with different numbers. In particular, we observed differences in brain activity between the numbers 1, 2, 3, 4, and the number 5, in terms of the shape and amplitude of the signal. In addition, we identified differences between digits and counting white circles on a black background. This is the first time that such differences have been observed. The methods developed for this project can be used in further MEG data studies to study the perception of the number in the brain.

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1. Background.

1.1. The MEG. The recording device we used, is the MEG (Magnetoencephalography) machine. This functional neuroimaging device uses very sensitive magnetometers to map brain activity by recording magnetic fields induced by electrical currents occurring naturally in the brain. The human brain is one of the most complex yet organized structures. There are at least 10^{10} neurons in the cerebral cortex, which includes 10^{14} interconnections between them (synapses) [4]. When information is processed in the brain, small currents flow in the neural system due to changes in membrane charge (potential). Usually, one magnetic field is measured around the scalp in 10fT (Femto-Tesla), which is a very low magnitude scale. For comparison, the average magnitude of the magnetic fields in an urban environment is measured at a scale of 100 million fT. Therefore, about 50,000 neurons should fire simultaneously in the same direction to detect brain activity. The MEG machine records brain data in analog form and the digitization is analog to digital (AD) conversion.

In our experiment, the subjects are under the MEG device, looking at a projector screen, reacting, and the MEG is recording the magnetic field related to the brain activity. Synchronization between the MEG device and the subject's brain is done by the digitizing operation, which is the most important step in the preparation of the experiment.

The projectors can be of different types such as liquid crystal display (LCD) or digital light processing (DLP). Usage of different projections may affect the results. In some projectors, the onsets of different colors may differ, and the timing may not be precise enough. For example, DLP-based projectors are less likely to have timing and luminance issues. Moreover, DLP-based projectors present the whole frame at once, and thus the timing is precisely the same on any location of the screen.

Various screens, such as cathode-ray tubes and monitors, and video projectors, can be utilized; some of these latter video projectors can have approximately 20 to 30 ms (milliseconds) lags or rise times and interlacing issues, which may have a significant effect on the lateness of evoked responses. In the MEG environment, visual stimuli are often presented on a back-projection screen, so that all magnetic parts of the stimulation system can be kept outside the shielded room.

1.2. Numeration. The average amount of numbers that humans encounter in one hour can vary between hundreds and thousands, not only in the classic representation like the written word or digital presentation but also via the counting of objects, hearing, etc. Researchers have discovered that humans had a numerical perception even in ancient times, which was used for daily decisions, such as searching for the tree that has the largest number of fruit, or detecting approaching danger [5].

The main question that still remained open is how the concept of the number is represented in the human brain. It may differ from counting, which is a skill that even children master quickly. The understanding of the concept of a very large quantity is often difficult for adults [6]. There are two main schools of thought about numerical perception; those who believe that the perception of a number is distinct for each number, and those who believe that the number is a representation of a magnitude in the brain and is a repetitive process done to a specific number. Numerical perception research that has been performed on apes indicates that there are regions inside the brain of apes that were more active for some numerical stimuli presented to the apes compared to other brain regions. Moreover, evidence of humans from the 20th century has shown that head injuries and trauma in specific areas of the brain resulted in a lack of functioning concerning basic arithmetic skills. Areas common to numbers in various forms include core visual and oculomotor areas, fusiform gyri, inferior frontal gyri, cingulate gyrus, insula, cerebellum, superior parietal lobule, and inferior parietal lobule. In the core visual areas (e.g., inferior and middle occipital gyri), as in most of the paradigms, the stimuli were visually presented. Similarly, eye movements present during visual tasks generate saccades [1]; [2], which elicit concordant activity in the middle and superior frontal and precentral gyri. This significant accord among studies on core visual and oculomotor regions was expected and serves as a calibration measure of our technique.

2. The experiment and data acquisition.

2.1. The Process of data collecting and preparing. MEG recordings were conducted on 32 subjects (see Section 2.2), using a sampling rate of 1017 Hz, online 1- to 400-Hz band-pass filter with a whole-head, 248-channel magnetometer array (4-D Neuroimaging, Magnets 3600WH) in a supine position, inside a magnetically shielded room. Reference coils located a short distance ($\sim$ 30 cm) away from the 248 sensors and oriented in the x, y, and z axes were used to record environmental noise. Head position was indicated by attaching five coils to the scalp and determining their positions in relation to the sensor array (see the digitization in Section 2.3). Visual stimulates were projected to the subject (see Section 2.4), questions were displayed, and brain signals were recorded during the display time and response time. The question ended when the subject pressed one of the 5 buttons corresponding to his answer. This data was stored in the MEG computer, was cleaned (Section 3.1), and followed by an analysis (Section 3).

2.2. The Subjects Thirty-two volunteered to participate in the experiment. Subjects were aged between 18 and 35 years (M = 24.15, STD = 4.021). All had normal or corrected to normal vision reported, and the majority were reported to be right-handed. We did not include data from subjects who gave more than 40 wrong answers (i.e., experiments that contained more than 20% errors), assuming that they did not understand the task correctly. This exclusion criterion was set during the data collection and was selected based on our work using the IEM method for tracking locations stored in the working memory [3]. Six subjects were dropped due to this criterion. The data from five additional subjects was excluded due to an error in stimulus presentation (Display diode was not set). Thus, the final analysis included 21 subjects who provided data on at least 160 stimuli. We ensured that the subject's entire head was inside the helmet in addition to their field of view being clear. The screen was perpendicular to the field of view. In Figure 1, we see the subject having her brain recording taken.

Fig. 1. Subject inside the MEG machine

2.3. Digitization. The positioning of the sensors around the subject head was done according to the specification of the MEG device in our lab (4-D Neuroimaging, Magnets 3600WH). In Figure 2 we see that the map consists of 248 sensors that are homogeneously scattered across the scalp. In Figure 3 we see the map of the active sensors. To see the activity of a specific sensor, we pressed it with the cursor. In Figure 4 we see the activity of the specific sensor. In Figure 5 we see the topography of the activity areas.

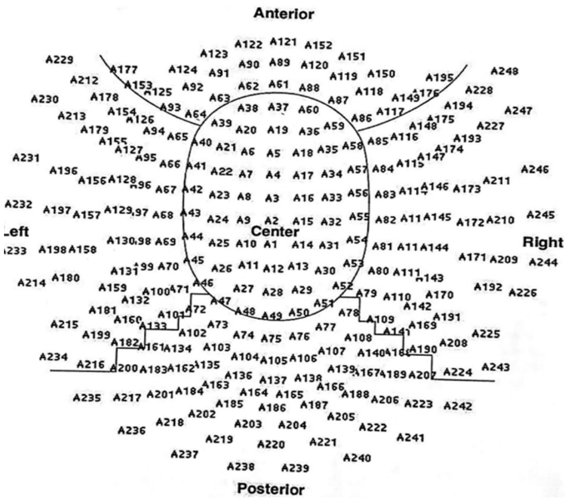

Fig. 2. The map of the sensors, we see 248 electrodes scattered across the skull

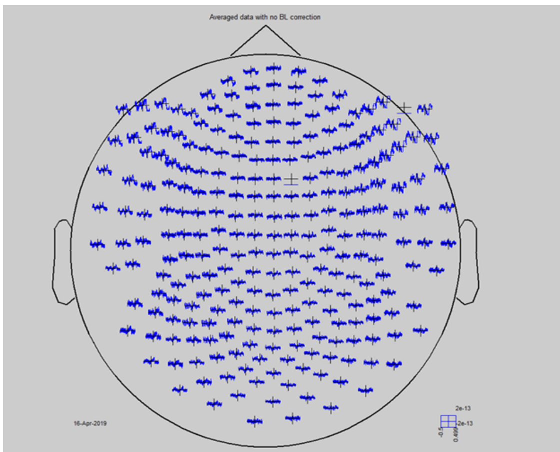

Fig. 3. In this screen you can click on each of the 248 sensors individually and see the behavior of the area it covers, as seen in Figure 2

2.4. Visual Stimulation. For visual stimulation while the subjects are in the MEG device, we used a projector with a rapid shutter that produces instantaneous stimulus delivery: Hitachi CP-SX635 projector with an ultra-long throw lens outside the magnetically-shielded room. The image is projected via a set of 2-mirrors to a screen (43 cm × 34 cm) approximately 60 cm from the

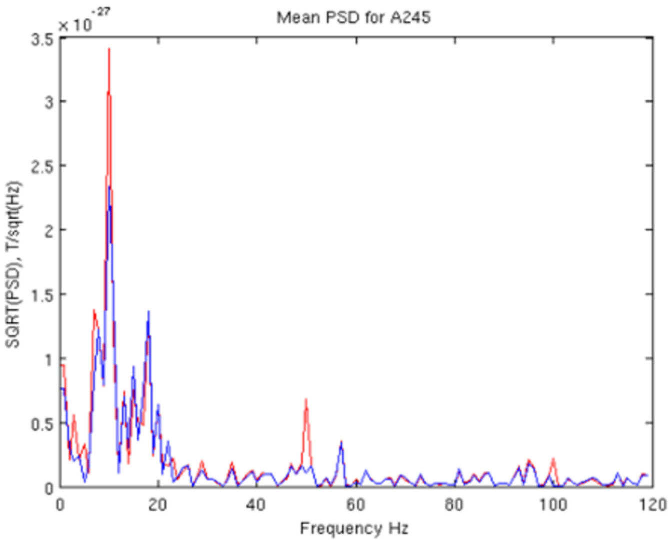

Fig. 4. Computing the result values obtained from one electrode is done with MATLAB and Fieldtrip libraries. Here we see the power spectrum

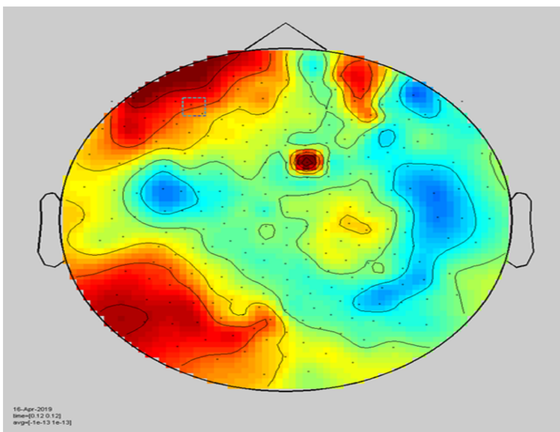

Fig. 5. The active areas are marked using colors (a dark color indicates high activity). Using Figure 4, we can sort the activated areas by the number of electrodes. This helps us to analyze the activity and the results

subject. It is important to mention that the photocell-equipped device that we used ensured an accurate measure of stimulus timing, especially the onset and rise time of the stimulus on the screen, relative to the timing of the stimulus onset in the stimulus-delivery software. Attention was paid to avoid abrupt luminance changes on the screen for stimulus onsets. For example, the background display was set between the stimuli to the mean overall luminance of the stimuli. We monitored the distance of stimuli from the subject's eyes and the two-dimensional size of the image, since this affects brain responses. When needed, other stimulus properties, such as the overall stimulus luminance, contrast, and spatial frequency were quantified, for instance, by using image-processing software during stimulus creation.

3. Data analysis.

3.1. Cleaning process. Firstly, external noise (e.g., powerline or mechanical vibrations) as well as the subjects heartbeat were removed from the data using Tal [8] and Shapiras [7] algorithm. Signal preprocessing and analysis at the sensor level were then implemented with the FieldTrip toolbox in MATLAB. Next, the data was segmented into 2,000 milliseconds (using epoch), starting at 500 milliseconds prior to the stimulus presentation and ending at 1,200 milliseconds following the stimulus presentation. Each data epoch was visually inspected for muscle artifacts and power jumps. These were removed from the data by subtracting the mean amplitude of the pre-stimulus period of 300 milliseconds before each problematic trial from all the data points in the segment. Furthermore, CreateCleanFile was implemented to clean data from lab disturbances, such as the air-conditioning frequency and elevator movements. Finally, spatial independent component analysis was applied to identify and remove eye movements and remaining potential contamination from the data.

3.2. Data shrinking. To analyze the results of our experiment, we organized the output for each participant obtained from the MEG device in tables such as the one in Figure 6. Each row marks a timestamp, and each column marks the sensor number from which we obtain the information. This table indicates the stimulus and response as numbers which imply the strength of the magnetic field in a particular sensor.

Next, we performed the data shrinking algorithm as follows.

1. We isolate the triggers for which the participant provided the correct response, and reject the incorrect triggers.

2. We perform principal component analysis (PCA), a mathematical method of decreasing dimensions while maintaining a high percentage (over 90%)

	sensors						
	0	1	2	.....	247	Trigger	Response
time							

Fig. 6. Table 1

of the original information. In our experiment, we applied PCA to decrease the dimension of the data from 248 to 10.

3. We add the NORM column to the table to obtain the table in Figure 7. The NORM represents the standard norm of the sample's ten components of PCA together for each row.

4. We extract peaks from the NORM column using a Gaussian filter over each component. This process is described in the next section.

	PCA COMPONENTS								
	0	1	2	3	...	9	Trigger	Response	NORM
t1									
t2									
...									
tn									

Fig. 7. Table 2

3.3. Gaussian filter and peak extraction. In signal processing, a Gaussian filter is a filter whose impulse response is a Gaussian function. Gaussian filters can be applied to the input surface by convolving the measured surface with a Gaussian weighting function. Such functions have no overshoot to a step function input while minimizing the rise and fall time. It is considered to be an ideal time domain filter, just as is the ideal frequency domain filter.

One of the challenges we encountered when we optimized the data was the peak pick problem. As illustrated in Figure 8, occasionally a peak is formed in the graph followed immediately by a drop, and again the graph rises to a new peak. By applying a local max function, we calculate the width and relative height of each peak (Figure 8, middle), and filter out irrelevant peaks. We remain with the significantly wider peaks, which maintain as much reliable information as possible. Of course, the width of the wider peaks will correspond to the amount of irrelevant data. (Figure 8).

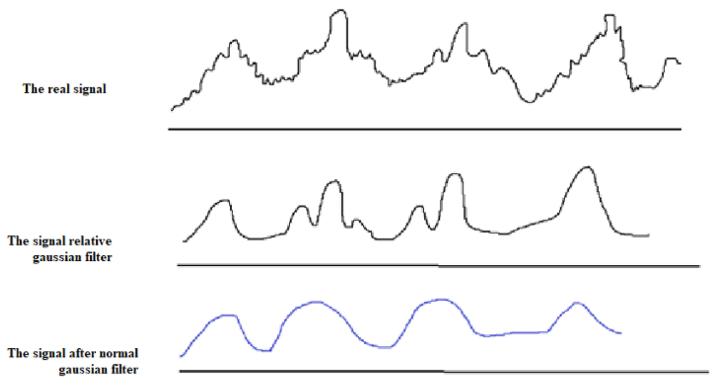

Fig. 8. Gaussian filter

When we discuss relative height, we are referring to height in relation to our MEG environment (and not to height in relation to a general reference point along the entire timeline (Figure 8, bottom)). If the graph is on the rise, and there was no activity because of a stimulus, then we obtain that all time points are relevant since they are all above the registered threshold. This may produce unreliable data which contains irrelevant information.

3.4. Principal component analysis. After performing PCA (principal component analysis) and the peak extraction processes, we obtain a set of data consisting of the most significant components, representing a combination of channels. The data is presented as points in a 10-dimensional vector space where each component is an axis (coordinate). In our results, we display the first 3 coordinates (in the order of the PCA variance importance) of the records in the extracted peaks respective to the stimuli (Figures 9, 11, 12, 13). Note that the new coordinate system does not represent the geometry of the brain, rather it is of an abstract space. Therefore, the coordinate of each point in the abstract space is the measurement of the magnetic field in some channels which are combined at that time. If two points are close in the abstract space, then these points are "close" measurements in terms of the magnitude that sampled at the most significant channels (in the PCA components variance importance order).

4. Results.

4.1. Brain activity while observing digits 1 to 5. The first experiment studies how the brain reacts to digits: the screen displays the digits 1 to 5, and the MEG collects brain data from the participant's reaction between observing the number and pressing the button. We then process the data as described above, and obtain graphs which contain all the records extracted out of peak moments, with different colors respective to the stimuli. Each color symbolizes a digit according to the color key of the corresponding button in the participant's right hand (see Section 2.1). Each point on the graph symbolizes the magnitude of the magnetic field and the duration of the response to the stimulus.

As can be seen in Figure 9, taken from an individual participant, the pink and orange points (representing the digits 3 and 4) are spread over more space than the blue and yellow points (representing the digits 1 and 5). We conclude from the graph that when the participant observes the digit 3 or the digit 4, they invest more brain effort in comparison to the digits 1 or 5 and in their turn, they invest much more effort than in the digit 2. Lastly, we can arrange the data in a bar graph which roughly follows a Gaussian curve (see Figure 8).

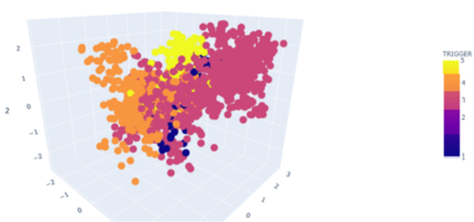

Fig. 9. Digits 1, 2, 3, 4, 5

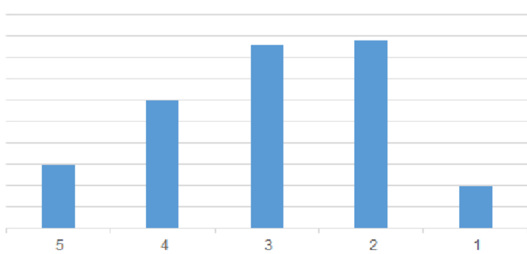

Fig. 10. Distribution of magnetic field strength
by number roughly follows a Gaussian curve pattern

4.2. Brain activity while counting circles in comparison to observing digits. The second experiment analyzes the difference in brain data upon observing numbers with different stimulus, as a digit (1 to 5) versus as a number of white circles. The results are displayed below: the blue points represent the digit stimulus, and the yellow points represent the white circles stimulus. As shown in Figure 11, Figure 12 and Figure 13, we notice that both forms of stimuli share a center in the abstract graph space. However, there is greater brain activity corresponding to the white circles representation.

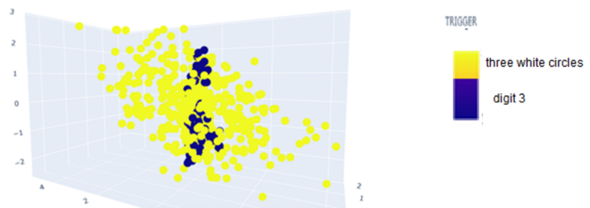

Fig. 11. The number 3 in two different forms

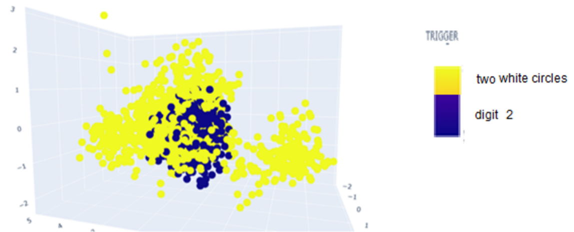

Fig. 12. The number 2 in two different forms

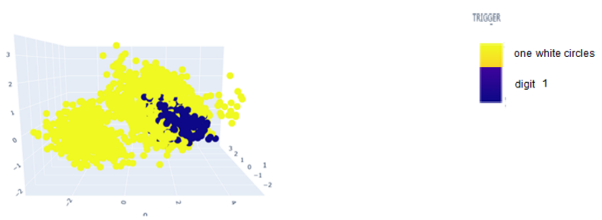

Fig. 13. The number 1 in two different forms

5. Discussion. For many years researchers have been researching the difference between two general types of mathematical thinking – geometric vs. algebraic. The original motivation of the present research was to use brain imaging to distinguish between the two. In other words, does a person with a geometric/algebraic tendency have a pattern of a particular brain activity? What distinguishes this pattern from other patterns of activities? This led us to pose a hypothesis that there is an abstract concept of number in the brain, the verification of which is a great challenge in itself.

We designed and conducted this experiment in order to find different representations of a number in the brain. We asked whether representing a number as a set of white circles is different from the standard figure of the number. In addition, we looked for possible differences between the numbers themselves. For example, we investigated whether the areas in the brain of a person who sees the number 4 respond in the same way as when this person sees the numbers 1, 2, 3, and 5. We arrived at the following conclusions after conducting the experiment and processing the results: The differences in the subject's brain response to a stimuli of a white circles, in contrast to stimuli of a digit (corresponding to the number of circles), pinpoint the different efforts of the subject. We were surprised to observe that when the subject noticed a single circle, there was a greater effort than when noticing the digit 1. Thus, the assumption that the difference was due to the effort that the subject makes in counting the circles was refuted by the experiment on one circle. When we analyzed the brain response to a digit-type stimulus, and compared all the digits to each other, we noticed that for the digits 2,3,4, the brain response was more active compared to seeing the digits 1 and 5. This indicates the subject's excitement when observing one of the middle digits. Moreover, the different digits have different brain reactions which can be observing from the axis points. In running the experiment comparing the concept of number, when represented by white circles vs. a digit, we discovered that the two representations share a common center in the digitization map, and each axis points to a specific component of the brain activity and its intensity. This supports the hypothesis that there is a theoretical concept of number in the brain.

6. Speculations (Expectations). This research focused on processing the concept of number and identifying the meaning that emerges from it. In our novel experiment design, for the first time, higher perceptual processes were studied directly (in contrast to interposing abnormal number variations). After processing the data and analyzing the results, we arrived at the following conclusions:

- For different numbers, there are different response areas in the human brain. As expected, these areas are in the prefrontal cortices.
- For different representations of the same number, the center of the reaction area in the digitization map is the same, but the continuation of the response varies according to the type of representation of the number.
- The human brain uses more resources when it responds to a stimulus of the type of seeing a group of white circles, compared to a stimulus of seeing the same quantity in its numerical form.

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