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A SHORT NOTE ON THE WEDDERBURN COMPONENTS OF A SEMISIMPLE FINITE GROUP ALGEBRA

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Abstract. One of the classical problems in the subject of group algebras is that of deducing Wedderburn decomposition of a finite semisimple group algebra. In this short note, we discuss how to check whether a matrix ring over a finite field is a Wedderburn component of the Wedderburn decomposition of a group algebra or not. Finally, we formulate an open problem in this direction.

1. Introduction and Main results. Let G be a finite group. Let $\mathbb{F}_q$ be a Galois field containing $q = p^k$ elements, where p is a prime such that $p \nmid |G|$. Let $\mathbb{F}_q G$ denote the group algebra generated by $\mathbb{F}_q$ and G. We refer to [11] for a nice survey on group algebras. Since $p \nmid |G|$, the group algebra $\mathbb{F}_q G$ is semisimple. Therefore, the well-known Wedderburn-Artin theorem (see [11]) implies that

$$\mathbb{F}_q G \simeq \oplus_{i=1}^{t} M_{n_i}(\mathbb{F}_{q_i}). \tag{1}$$

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The above decomposition of $\mathbb{F}_q G$ is known as Wedderburn decomposition (WD). Here t is unknown, $\mathbb{F}_{q_i}$, for each i, is an unknown finite extension of $\mathbb{F}_q$ and n_i's are also unknowns. In the subject of semisimple group algebras, deducing the WD is a very important and extensively studied reserach problem (see [1, 2, 3, 4, 7, 12, 13, 14, 15, 16, 17, 18, 19]).

In the last few years, a lot of work has been done in the computation of WD of semisimple group algebras of non-metabelian groups (groups whose derived subgroup is non-abelian). This is because the study of WD of metabelian and normally monomial groups have been completed in [3, 4]. To the best of our knowledge, the largest non-metabelian groups (apart from symmetric and quaternion groups) studied till now for computing WD are that of order 144, 168, 360 (see [15, 2, 1]). For the larger groups, it is becoming more and more difficult to uniquely characterize the WD. To see this, we note from (1) using dimension formula that

$$|G| = \sum_{i=1}^{t} (n_i^2 \times [\mathbb{F}_{q_i} : \mathbb{F}_q]), \tag{2}$$

where $[\mathbb{F}_{q_i} : \mathbb{F}_q]$ denotes the degree of extension of $\mathbb{F}_{q_i}$ over $\mathbb{F}_q$. Let S be the exponent of the group G, i.e., the lcm of the orders of its elements. Clearly, $p \nmid S$. Over $\mathbb{F}_q$, let $\wp$ be the primitive S^{th} root of unity. Consequently, $\mathbb{F}_q(\wp)$ denotes the splitting over $\mathbb{F}_q$. Furthermore, we denote the set

$$\mathcal{T}_{\mathbb{F}_q} = \{d \mid \varphi(\wp) = \wp^d, \forall\ \varphi \in \mathrm{GL}(\mathbb{F}_q(\wp)/\mathbb{F}_q)\}.$$

Here $\mathrm{GL}(\mathbb{F}_q(\wp)/\mathbb{F}_q)$ represents the Galois group of $\mathbb{F}_q(\wp)$ over $\mathbb{F}_q$. Using [8, Theorem 2.21], one can characterize the set $\mathcal{T}_{\mathbb{F}_q}$. More precisely, we have

$$\mathcal{T}_{\mathbb{F}_q} = \{1, q, \ldots, q^{y-1}\} \mod S, \tag{3}$$

where y is the order of q. For a p'-element $a \in G$, we define $\gamma_g = \sum_{g_1 \in \mathfrak{C}_g} g_1$. Here $\mathfrak{C}_g$ denotes the conjugacy class of g in G. Then, the cyclotomic $\mathbb{F}_q$-class of γ_g can be represented as

$$\mathfrak{S}_{\mathbb{F}_q}(\gamma_g) = \{\gamma_{g^d} \mid d \in \mathcal{T}_{\mathbb{F}_q}\}. \tag{4}$$

Next, we recall that $[\mathbb{F}_{q_i} : \mathbb{F}_q]$ and t can be deduced by computing the cyclotomic $\mathbb{F}_q$-classes of the group G (see Propositions 1 and 2). Therefore, it remains to deduce the exact Wedderburn components (or values of n_i's) in (2). This is a

very uphill task as the size of the group increases. To understand this statement clearly, let us consider the example of the only non-metabelian group G of order 150. It can be shown that the group $G \simeq (C_5 \times C_5) \rtimes S_3$, where S_3 is a symmetric group on 3 symbols and C_5 is a cyclic group of order 5, is non-metabelian since its derived subgroup is $G' \simeq (C_5 \times C_5) \rtimes C_3$, which is non-abelian. Also, this group has 13 conjugacy classes. Let S be the exponent of the group G, which is 30. Also, let p be such that $p^k \equiv 1 \mod 30$. Before proceeding further, let us recall a result from [6].

Proposition 1 ([6, Proposition 1.2])**.** *The simple components of a semisimple group algebra $\mathbb{F}_qG$ are in* 1-1 *correspondence with the set of cyclotomic $\mathbb{F}_q$-classes in G.*

Since the Galois group of any finite field is cyclic, [6, Theorem 1.3] gives:

Proposition 2. *Let L be the number of cyclotomic $\mathbb{F}_q$-classes in G. If $\kappa_1, \kappa_2, \cdots, \kappa_L$ denote the simple components of the center of semisimple group algebra $\mathbb{F}_qG$ and $\mathfrak{S}_1, \mathfrak{S}_2, \cdots, \mathfrak{S}_L$ are the cyclotomic $\mathbb{F}_q$-classes of G, then $|\mathfrak{S}_i| = [\kappa_i : \mathbb{F}_q]$ for every i with a suitable ordering of the indices.*

It follows from (3) that for the group G, we have

$$\mathcal{T}_{\mathbb{F}_q} = \{1, q, \ldots, q^{y-1}\} \mod 30 = \{1\}$$

as $p^k \equiv 1 \mod 30$. Consequently, (4) derives that $\mathfrak{S}_{\mathbb{F}_q}(\gamma_g) = \{\gamma_g\}$ for any $g \in G$. This along with Proposition 2 yields that $1 = |\mathfrak{S}_i| = [\kappa_i : \mathbb{F}_q]$. In other words, we must have $\kappa_i = \mathbb{F}_q$ for each i or $\mathbb{F}_{q_i} = \mathbb{F}_q$ in (2) for each i. As G has 13 conjugacy classes, Proposition 1 confirms that t in (2) is 13. Therefore, at this point, we rewrite (1) and (2) as

$$\mathbb{F}_qG \simeq \oplus_{i=1}^{13} M_{n_i}(\mathbb{F}_q), \text{ where } 150 = \sum_{i=1}^{13} n_i^2. \tag{5}$$

Next, using the normal subgroups of G, one can easily compute several n_i's appearing in (5). To see this, we need the following result. Its proof can be found in [11].

Proposition 3. *Let $\mathbb{F}_qG$ be a semisimple group algebra and N be a normal subgroup of G. Then $\mathbb{F}_qG \cong \mathbb{F}_q\big(G/N\big) \oplus \Delta(G, N)$, where $\Delta(G, N)$ denotes an ideal of $\mathbb{F}_qG$ generated by the set $\{n - 1 : n \in N\}$. Furthermore, if $N = G'$ (commutator subgroup), then every component in the decomposition of $\Delta(G, N)$ must be non-commutative.*

We already know that $G' \simeq (C_5 \times C_5) \rtimes C_3$, which means $G/G' \simeq C_2$. Also, it is straight-forward to note that $\mathbb{F}_q C_2 \simeq \mathbb{F}_q \oplus \mathbb{F}_q$. Therefore, this and Proposition 3 along with (5) imply that

$$\mathbb{F}_q G \simeq \mathbb{F}_q^2 \oplus_{i=1}^{11} M_{n_i}(\mathbb{F}_q), \text{ where } 148 = \sum_{i=1}^{11} n_i^2,\ n_i \geq 2. \tag{6}$$

The important thing about taking $N = G'$ is that it limits $n_i \geq 2$ in (6), which is $n_i \geq 1$ in (5), since every component in the decomposition of $\Delta(G, G')$ must be non-commutative. Next, by construction, we note that G has another normal subgroup N_1 isomorphic to $C_5 \times C_3$, where $G/N_1 \simeq S_3$. At this stage, we recall from [22] that $\mathbb{F}_q S_3 \simeq \mathbb{F}_q^2 \oplus M_2(\mathbb{F}_q)$. Substituting this in (6) after applying Proposition 3 with $N = N_1$ to reach at

$$\mathbb{F}_q G \simeq \mathbb{F}_q^2 \oplus M_2(\mathbb{F}_q) \oplus_{i=1}^{10} M_{n_i}(\mathbb{F}_q), \text{ where } 144 = \sum_{i=1}^{10} n_i^2,\ n_i \geq 2. \tag{7}$$

To this end, we emphasize that Proposition 3 can no longer be helpful in further determining the remaining n_i's in (7) as G has no other non-trivial normal subgroups except the two discussed above. We note that (7) have 12 possible choices of n_i's fulfilling $144 = \sum_{i=1}^{10} n_i^2,\ n_i \geq 2$. These are

$$(2^7, 4, 6, 8),\ (2^5, 3^3, 4, 9),\ (2^5, 3, 4, 5^2, 7),\ (2^4, 3^3, 4, 6, 7),$$

$$(2^3, 3^4, 4^2, 8),\ (2^3, 3, 4^3, 5^3),\ (2^3, 4^6, 6),\ (2^2, 3^4, 5^4),$$

$$(2^2, 3^3, 4^3, 5, 6),\ (2, 3^6, 5^2, 6), (3^8, 6^2), (3^7, 4^2, 7). \tag{8}$$

Next, we recall an important result from [5, Proposition 1]. This result shows that if $\mathbb{F}_q G \simeq \oplus_{i=1}^{t} M_{n_i}(\mathbb{F}_{q_i})$, then $p \nmid |n_i|$ for any i. If $p = 7$, then $p^7 \equiv 1 \mod 30$, (i.e., $q = p^4$ in our case), which means that 7 must not appear in WD of the group algebra $\mathbb{F}_q G$. Consequently, the 12 choices in (8) are reduced to the following 9 choices:

$$(2^7, 4, 6, 8),\ (2^5, 3^3, 4, 9),\ (2^3, 3^4, 4^2, 8),\ (2^3, 3, 4^3, 5^3),\ (2^3, 4^6, 6),$$

$$(2^2, 3^4, 5^4),\ (2^2, 3^3, 4^3, 5, 6),\ (2, 3^6, 5^2, 6), (3^8, 6^2). \tag{9}$$

Finally, we observe that Propositions 1–3 and [5, Proposition 1] are no longer useful for uniquely deducing the Wedderburn components in (7). At this stage, we invoke a very important result from [12, Lemma 2.1] that will be enough to uniquely identify the Wedderburn components of a group algebra.

Lemma 4. *Let A_1 and A_2 denote the semisimple algebras having finite dimensions over $\mathbb{F}_q$. Further, let Ψ be an onto map between A_1 and A_2, then we must have*

$$A_1 \simeq A_3 \oplus A_2,$$

where A_3 denotes a semisimple $\mathbb{F}_q$-algebra.

If we show that $M_3(\mathbb{F}_q)$ and $M_6(\mathbb{F}_q)$ must be the Wedderburn components of $\mathbb{F}_qG$, and $M_5(\mathbb{F}_q)$ must not be the Wedderburn component of $\mathbb{F}_qG$, then we are sure from (9) that $(3^8, 6^2)$ is the only choice of n_i's that fulfils this condition. In this regard, Lemma 4 can be very useful. For this, if one can show that the maps $\Psi_1 : \mathbb{F}_qG \to M_3(\mathbb{F}_q)$ and $\Psi_2 : \mathbb{F}_qG \to M_6(\mathbb{F}_q)$ are onto algebra homomorphisms, then Lemma 4 guarantees that $M_3(\mathbb{F}_q)$ and $M_6(\mathbb{F}_q)$ must be the Wedderburn components of $\mathbb{F}_qG$. Moreover, if one can show that the map $\Psi_3 : \mathbb{F}_qG \to M_5(\mathbb{F}_q)$ is not an onto algebra homomorphisms, then Lemma 4 guarantees that $M_5(\mathbb{F}_q)$ must not be the Wedderburn component of $\mathbb{F}_qG$.

Thus, the main problem of this paper is discussed as follows:

Problem 5. *Let G be a finite group and let $\mathbb{F}_q$ be a field with $q = p^k$ elements, where p is a prime. Let Ψ be an algebra homomorphism, where*

$$\Psi : \mathbb{F}_qG \to M_n(\mathbb{F}_{q^z}),$$

$z \geq 1$ and $\mathbb{F}_{q^z}$ represents an extension field of degree z over $\mathbb{F}_q$. Then find out the values of n for which Ψ is onto.

It is trivial to note that $n \leq \sqrt{\frac{|G|}{z}}$ for Ψ to be onto. By [5, Proposition 1], we know that $p \nmid n$. Therefore, if n is a multiple of p, then the map Ψ is never onto. Consequently, a partial answer to Problem 5 is known. Furthermore, if $z > 1$, then we know the following important result.

Lemma 6 ([23])**.** *Let p and p' be two primes. Let $\mathbb{F}_q$ be a field with $q = p^{k_1}$ elements and let $\mathbb{F}_{q'}$ be a field with $q' = (p')^{k_2}$ elements, where $k_1, k_2 \geq 1$. Let both the group algebras $\mathbb{F}_qG, \mathbb{F}_{q'}G$ be semisimple. Suppose that*

$$\mathbb{F}_qG \cong \oplus_{i=1}^{t} M(n_i, \mathbb{F}_q),\ n_i \geq 1$$

and $M\left(n, \mathbb{F}_{(q')^z}\right)$ *is a Wedderburn component of the group algebra* $\mathbb{F}_{q'}G$ *for some* $z \geq 1$ *and any positive integer* n*, i.e.,*

$$\mathbb{F}_{q'}G \cong \oplus_{i=1}^{s-1} M(m_i, \mathbb{F}_{q_i'}) \oplus M(n, \mathbb{F}_{(q')^z}),\ m_i \geq 1.$$

Here $\mathbb{F}_{q_i'}$ *is a field extension of* $\mathbb{F}_{q'}$*. Then* $M\left(n, \mathbb{F}_q\right)$ *must be a Wedderburn component of the group algebra* $\mathbb{F}_qG$ *and it appears atleast* z *times in the Wedderburn decomposition of* $\mathbb{F}_qG$*.*

The above lemma clearly indicates that for $z > 1$, if Ψ in Problem 5 is onto, then whenever $\mathbb{F}_{\bar{q}}$ acts as a splitting field for some prime power $\bar{q}$, the map

$$\Psi' : \mathbb{F}_{\bar{q}}G \to M_n(\mathbb{F}_{\bar{q}})$$

must be onto. In other words, Lemma 6 guarantees that the dimensions of matrix algebras over extensions of the base field must be the multiples of the dimensions of matrix algebras over the base field. This means, for $z > 1$, one can categorize the possible values of n for which the map Ψ in Problem 5 is not onto by looking at the values of n whenever $z = 1$. Therefore, in the rest of the paper, we always assume that $z = 1$.

In the direction of fully solving Problem 5, we give a conjecture and discuss certain examples in the support of our conjecture. Before proposing the conjecture, we discuss an important notation. Let $\Phi : G \to GL_{n-1}(\mathbb{F}_q)$ be a homomorphism, where $n \geq 3$. For any $a \in G$, let Φ^* be a lifting map $\Phi^* : G \hookrightarrow GL_n(\mathbb{F}_q)$ defined as

$$\Phi^*(a) = \begin{bmatrix} \phi(a) & 0 \\ 0 & 1 \end{bmatrix}.$$

Conjecture 7. *Let* G *be a finite non-abelian group and let* $\mathbb{F}_q$ *be a field with* $q = p^k$ *elements, where* p *is a prime. Suppose* $\Phi' : G \to GL_n(\mathbb{F}_q)$ *is a group homomorphism such that* Φ' *is one-one and* Φ' *is not a lifting map. Then the map* Φ' *can be extended to an onto algebra homomorphism between* $\mathbb{F}_qG$ *and* $M_n(\mathbb{F}_q)$*.*

Let us first see what happens if Φ' is a lifting map in Conjecture 7. If Φ' is a lifting map, then $\Phi'(a) = \begin{bmatrix} \phi''(a) & 0 \\ 0 & 1 \end{bmatrix}$ for every $a \in G$, where $\Phi'' : G \to GL_{n-1}(\mathbb{F}_q)$ is a homomorphism. In this situation, we note that the element $\begin{bmatrix} \Phi''(a) & 1 \\ 0 & 1 \end{bmatrix} \in M_n(\mathbb{F}_q)$ for any $a \in G$ have no pre-image under any extension of Φ' as a homomorphism between $\mathbb{F}_qG$ and $M_n(\mathbb{F}_q)$. This is because on adding/multiplying any

two elements a and b in the group algebra $\mathbb{F}_qG$, $\Phi'(a+b)/\Phi'(a*b)$ is always of the form $\begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}$, where r_1 and r_2 are elements of $\mathbb{F}_q$. Thus, Φ' can never be onto.

Next, in the support of our conjecture, we discuss few examples. Let us begin with S_3 and $q = 5$. We know that $S_3 = \{x, y : x^2 = y^3 = 1, xyx^{-1} = y^2\}$. To define any map from S_3, it is enough to define it on x and y, as they are the generators of S_3. Let us define $\Phi' : S_3 \to GL_2(\mathbb{F}_5)$ as

$$\Phi'(x) = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}, \ \Phi'(y) = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}.$$

One can note that the map Φ' is one-one homomorphism. This map can be linearly extended to onto algebra homomorphism between $\mathbb{F}_5S_3$ and $M_2(\mathbb{F}_5)$. Our argument is supported by [22] as $M_2(\mathbb{F}_5)$ is a Wedderburn component of $\mathbb{F}_5S_3$. However, if we define a lifting Φ^* of Φ' as

$$\Phi^*(x) = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ \Phi^*(y) = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

In this case, although Φ^* is one-one, but it cannot be linearly extended to onto algebra homomorphism between $\mathbb{F}_5S_3$ and $M_3(\mathbb{F}_5)$, since $M_3(\mathbb{F}_5)$ is not a Wedderburn component of $\mathbb{F}_5S_3$.

Our next example is related to alternating group A_5. We know that $A_5 = \{x, y : x^3 = y^2 = (yx)^5 = 1\}$ and we take $p = 5$ (here we are not considering semisimple group algebra). Let us define $\Phi' : A_5 \to GL_3(\mathbb{F}_5)$ as

$$\Phi'(x) = \begin{bmatrix} 4 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 0 \end{bmatrix}, \ \Phi'(y) = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 4 & 4 \\ 0 & 0 & 4 \end{bmatrix}.$$

One can note that the map Φ' is one-one homomorphism. This map can be linearly extended to onto algebra homomorphism between $\mathbb{F}_5A_5$ and $M_4(\mathbb{F}_5)$. This is true as $M_3(\mathbb{F}_5)$ is a Wedderburn component of $\mathbb{F}_5A_5$ (see [10]).

Next, we discuss about the Wedderburn components of the group algebras of groups $SL_2(\mathbb{F}_p)$ for primes $3 \leq p \leq 7$ and A_n for $6 \leq n \leq 7$ (the group algebra of the group A_5 is already discussed above). For $G = SL_2(\mathbb{F}_3)$, one can see that there exists a one-one map from G to $GL_2(\mathbb{F}_5)$ as the matrices

$$\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 1 & 2 \end{bmatrix}$$

generates a subgroup in $SL_2(\mathbb{F}_3)$ isomorphic to G. This means by Conjecture 7, $M_2(\mathbb{F}_5)$ is a Wedderburn component of $\mathbb{F}_qG$, which is true (see [9]). Similarly, we can show that $M_3(\mathbb{F}_5)$ is a Wedderburn component of $\mathbb{F}_qG$ by using our conjecture.

For $G = SL_2(\mathbb{F}_5)$, one can see that there are no one-one maps from G to $GL_2(\mathbb{F}_5)$ and G to $GL_3(\mathbb{F}_5)$. In this case, Conjecture 7 is not applicable. However, we note that $M_2(\mathbb{F}_5)$ and $M_3(\mathbb{F}_5)$ are not the Wedderburn components of $\mathbb{F}_qG$, which is true (see [21]). Also, we can show that $M_4(\mathbb{F}_5)$ is a Wedderburn component of $\mathbb{F}_qG$ by using our conjecture.

For $G = SL_2(\mathbb{F}_7)$, one can see that there is no one-one map from G to $GL_2(\mathbb{F}_{11})$. Consequently, $M_2(\mathbb{F}_{11})$ is not the Wedderburn component of $\mathbb{F}_qG$ (this result is verified through GAP). Also, we can show that $M_3(\mathbb{F}_{11})$ and $M_4(\mathbb{F}_{11})$ are Wedderburn components of $\mathbb{F}_qG$ by using our conjecture.

For $G = A_6$ and $p = 7$, one can see that there are no one-one maps from G to $GL_2(\mathbb{F}_7)$ and G to $GL_3(\mathbb{F}_7)$. Also, [2] implies that $M_2(\mathbb{F}_7)$ and $M_3(\mathbb{F}_7)$ are not the Wedderburn components of $\mathbb{F}_qG$. Further, we can show that $M_5(\mathbb{F}_7)$ is the Wedderburn component of $\mathbb{F}_7G$ by using our conjecture.

For $G = A_7$ and $p = 11$, one can see that there are no one-one maps from G to $GL_2(\mathbb{F}_{11})$ and G to $GL_3(\mathbb{F}_{11})$. Also, it is confirmed through GAP that $M_2(\mathbb{F}_{11})$ and $M_3(\mathbb{F}_{11})$ are not the Wedderburn components of $\mathbb{F}_{11}G$.

Finally, we end this section by discussing an important observation related to our conjecture. We remark that the conditions given in the conjecture are only sufficient but not necessary. This means that if there is no such one-one group homomorphism $\Phi' : G \to GL_n(\mathbb{F}_q)$, then still $M_n(\mathbb{F}_q)$ may be the Wedderburn component of $\mathbb{F}_qG$. For example, in case of $G = SL_2(\mathbb{F}_7)$, there is no one-one homomorphism from G to $GL_3(\mathbb{F}_{11})$, but still $M_3(\mathbb{F}_{11})$ is the Wedderburn component of $\mathbb{F}_{11}G$ (this is verified by GAP).

2. Discussion. We have discussed several ways to deduce the Wedderburn components of a semisimple group algebra. In this direction, we have formulated a conjecture and provided many examples in the support of our conjecture. A positive answer to this conjecture would give a new way of determining the Wedderburn components of a semisimple group algebra.

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