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STUDY OF A GENERALIZED SUBCLASS OF MEROMORPHIC FUNCTIONS

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ABSTRACT. This paper is concerned with a new subclass of meromorphic close-to-convex functions defined by means of subordination. Various properties of this class such as coefficient estimates, inclusion relationship, distortion property and radius of meromorphic convexity, are established. Some earlier known results follow as special cases.

1. Introduction. Let $\mathcal{A}$ denote the class of functions f of the form $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$, which are analytic in the open unit disc $E = \{z : |z| < 1\}$. Further, the class of functions $f \in \mathcal{A}$ and which are univalent in E , is denoted by $\mathcal{S}$. A function w which has expansion of the form $w(z) = \sum_{n=1}^{\infty} c_n z^n$ and satisfy

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the conditions $w(0) = 0$ and $|w(z)| \leq 1$, is called a Schwarz function. The class of Schwarz functions is denoted by $\mathcal{U}$.

Let f and g are two analytic functions in E , then f is said to be subordinate to g , if there exists a Schwarz function $w \in \mathcal{U}$ such that $f(z) = g(w(z))$. If f is subordinate to g , then it is denoted by $f \prec g$. Further, if g is univalent in E , then $f \prec g$ is equivalent to $f(0) = g(0)$ and $f(E) \subset g(E)$.

The well known classes $\mathcal{S}^*$ of starlike functions and $\mathcal{K}$ of convex functions are defined as follows:

$$\mathcal{S}^* = \left\{ f : f \in \mathcal{S}, \operatorname{Re} \left(\frac{zf'(z)}{f(z)} \right) > 0, \quad z \in E \right\}$$

and

$$\mathcal{K} = \left\{ f : f \in \mathcal{S}, \operatorname{Re} \left(\frac{(zf'(z))'}{f'(z)} \right) > 0, \quad z \in E \right\}.$$

The concept of close-to-convex functions was given by Kaplan [6]. A function $f \in \mathcal{A}$ is said to be in the class $\mathcal{C}$ of close-to-convex functions if there exists a function $g \in \mathcal{S}^*$ such that

$$\operatorname{Re} \left(\frac{zf'(z)}{g(z)} \right) > 0 (z \in E).$$

By $\mathcal{M}$, we denote the class of functions f of the form

$$f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n z^n,$$

which are meromorphic, analytic in $E^* = \{z : z \in \mathbb{C}, 0 < |z| < 1\}$ and single valued.

A function $f \in \mathcal{M}$ is said to be in the class $\mathcal{MS}^*$ of meromorphic starlike functions if it satisfies the condition

$$\operatorname{Re} \left(\frac{zf'(z)}{f(z)} \right) < 0, z \in E^*.$$

The class $\mathcal{MK}$ of meromorphic convex functions is given by

$$\mathcal{MK} = \left\{ f : f \in \mathcal{M}, \operatorname{Re} \left(\frac{(zf'(z))'}{f'(z)} \right) < 0, z \in E^* \right\}.$$

It is obvious that $f \in \mathcal{MK}$ if and only if $-zf'(z) \in \mathcal{MS}^*$.

By $\mathcal{MC}$, we denote the class of meromorphic close-to-convex functions. A function $f \in \mathcal{M}$ is called meromorphic close-to-convex function if there exists a function $g \in \mathcal{MS}^*$ such that

$$\operatorname{Re} \left(\frac{zf'(z)}{g(z)} \right) < 0, z \in E.$$

Seker [11] established the class $\mathcal{K}_s^{(k)}(\gamma)$ ($0 \leq \gamma < 1$) of close-to-convex analytic functions $f \in \mathcal{A}$ which satisfy the condition

$$\operatorname{Re} \left(\frac{z^k f'(z)}{g_k(z)} \right) > \gamma,$$

where

$$(1) \quad g_k(z) = \Pi_{\nu=0}^{k-1} \epsilon^{-\nu} g(\epsilon^\nu z) (\epsilon^k = 1; k \geq 1),$$

and $g \in \mathcal{S}^* \left(\frac{k-1}{k} \right)$.

Motivated by the class $\mathcal{K}_s^{(k)}(\gamma)$, Yi et al. [14] and Tang et al. [13] studied the classes $\mathcal{MK}^{(k)}(\gamma)$ and $\mathcal{MK}^{(k)}(\eta, \gamma)$ of meromorphic close-to-convex functions, respectively. The classes are defined as follows:

$$\mathcal{MK}^{(k)}(\gamma) = \left\{ f : f \in \mathcal{M}, \operatorname{Re} \left(\frac{-f'(z)}{z^{k-2}g_k(z)} \right) > \gamma \right\},$$

and

$$\mathcal{MK}^{(k)}(\eta, \gamma) = \left\{ f : f \in \mathcal{M}, \frac{-f'(z)}{z^{k-2}g_k(z)} \prec \frac{1 + (1-2\gamma)\eta z}{1 - \eta z} \right\},$$

where $g_k(z) = \Pi_{\nu=0}^{k-1} \epsilon^{-\nu} g(\epsilon^\nu z) (\epsilon^k = 1; k \geq 1)$ and $g \in \mathcal{MS}^* \left(\frac{k-1}{k} \right)$, $0 \leq \gamma < 1$, $0 < \eta \leq 1$ and $z \in E^*$.

Raina et al. [9] introduced the class of strongly close-to-convex functions of order β , as below:

$$\mathcal{C}'_\beta = \left\{ f : f \in \mathcal{A}, \left| \arg \left\{ \frac{zf'(z)}{g(z)} \right\} \right| < \frac{\beta\pi}{2}, g \in \mathcal{K}, 0 < \beta \leq 1, z \in E \right\}.$$

For $-1 \leq B < A \leq 1$, Janowski [5] introduced the class of functions in $\mathcal{A}$ which are of the form $p(z) = 1 + \sum_{k=1}^{\infty} p_k z^k$ and satisfy the condition $p(z) \prec \frac{1 + Az}{1 + Bz}$.

This class plays an important role in the study of various subclasses of analytic-univalent functions. As a generalization of Janowski's class, Polatoglu et al. [7] introduced the class $\mathcal{P}(A, B; \alpha)$ ($0 \leq \alpha < 1$), the subclass of $\mathcal{A}$ which consists of functions p such that $p(z) \prec \frac{1 + [B + (A - B)(1 - \alpha)]z}{1 + Bz}$. Also for $\alpha = 0$, the class $\mathcal{P}(A, B; \alpha)$ agrees with the class defined by Janowski [5].

Inspired by the above mentioned classes, now we define the following class which is to study in this paper.

Definition 1. Let $\mathcal{MK}^{(k)}(A, B; \alpha; \beta)$ denote the class of functions $f \in \mathcal{M}$ which satisfy the conditions,

$$\frac{-f'(z)}{z^{k-2}g_k(z)} \prec \left(\frac{1 + [B + (A - B)(1 - \alpha)]z}{1 + Bz} \right)^\beta, \quad -1 \leq B < A \leq 1, z \in E^*,$$

where $g(z) = \frac{1}{z} + \sum_{n=1}^{\infty} b_n z^n \in \mathcal{MS}^* \left(\frac{k-1}{k} \right)$, $0 \leq \alpha < 1, 0 < \beta \leq 1, -1 \leq B < A \leq 1$ and $g_k(z)$ is defined in (1).

Particularly

- (i) $\mathcal{MK}^{(k)}((1-2\gamma)\eta, -\eta; 0; 1) \equiv \mathcal{MK}^{(k)}(\eta, \gamma)$, the class studied by Tang et al. [13].
- (ii) $\mathcal{MK}^{(k)}(1-2\gamma, -1; 0; 1) \equiv \mathcal{MK}^{(k)}(\gamma)$, the class introduced by Yi et al. [14].

As $f \in \mathcal{MK}^{(k)}(A, B; \alpha; \beta)$, by definition of subordination, there exists a Schwarz function $w \in \mathcal{U}$ such that

$$(2) \quad \frac{-f'(z)}{z^{k-2}g_k(z)} = \left(\frac{1 + [B + (A - B)(1 - \alpha)]w(z)}{1 + Bw(z)} \right)^\beta.$$

In this paper, we study the coefficient estimates, inclusion relationship, distortion theorem and radius of meromorphic convexity for the functions in the class $\mathcal{MK}^{(k)}(A, B; \alpha; \beta)$. The results proved by various authors follow as special cases.

Throughout this paper, we assume that $-1 \leq B < A \leq 1, 0 \leq \alpha < 1, 0 < \beta \leq 1, 0 \leq \gamma < 1, 0 < \eta \leq 1, k \geq 1, k$ -an integer, $z \in E^*$.

2. Preliminary lemmas. For the derivation of our main results, we must require the following lemmas:

Lemma 1 ([2, 10]). *Let,*

$$(3) \quad \left(\frac{1 + [B + (A - B)(1 - \alpha)]w(z)}{1 + Bw(z)} \right)^\beta = (P(z))^\beta = 1 + \sum_{n=1}^{\infty} p_n z^n,$$

then

$$|p_n| \leq \beta(1 - \alpha)(A - B), n \geq 1.$$

Lemma 2 ([4]). *For $g(z) = \frac{1}{z} + \sum_{n=1}^{\infty} b_n z^n \in \mathcal{MS}^*$, we have*

$$|b_n| \leq \frac{2}{n+1}.$$

Lemma 3 ([9]). *Let $-1 \leq B_2 \leq B_1 < A_1 \leq A_2 \leq 1$, then*

$$\left(\frac{1 + A_1 z}{1 + B_1 z} \right)^\beta \prec \left(\frac{1 + A_2 z}{1 + B_2 z} \right)^\beta.$$

Lemma 4 ([8]). *If $g \in \mathcal{MS}^*$, then for $|z| = r, 0 < r < 1$, we have*

$$\frac{(1-r)^2}{r} \leq |g(z)| \leq \frac{(1+r)^2}{r}.$$

Lemma 5 ([13]). *For $g(z) = \frac{1}{z} + \sum_{n=1}^{\infty} b_n z^n \in \mathcal{MS}^* \left(\frac{k-1}{k} \right)$, then*

$$G_k(z) = z^{k-1} g_k(z) \in \mathcal{MS}^*.$$

Lemma 6 ([1, 2]). *If $P(z) = \frac{1 + [B + (A - B)(1 - \alpha)]w(z)}{1 + Bw(z)}$, $-1 \leq B <$*

$A \leq 1, w \in \mathcal{U}$, then for $|z| = r < 1$, we have

$$Re \frac{zP'(z)}{P(z)} \geq \begin{cases} -\frac{(A-B)(1-\alpha)r}{(1-[B+(A-B)(1-\alpha)]r)(1-Br)}, & \text{if } R_1 \leq R_2, \\ 2\frac{\sqrt{(1-B)(1-[B+(A-B)(1-\alpha)])(1+[B+(A-B)(1-\alpha)]r^2)(1+Br^2)}}{(A-B)(1-\alpha)(1-r^2)} \\ -\frac{(1-[B+(A-B)(1-\alpha)]Br^2)}{(A-B)(1-\alpha)(1-r^2)} + \frac{(A+B)-\alpha(A-B)}{(A-B)(1-\alpha)}, & \text{if } R_1 \geq R_2, \end{cases}$$

where $R_1 = \sqrt{\frac{(1-[B+(A-B)(1-\alpha)])(1+[B+(A-B)(1-\alpha)]r^2)}{(1-B)(1+Br^2)}}$ and
 $R_2 = \frac{1-[B+(A-B)(1-\alpha)]r}{1-Br}.$

3. Main results.

Theorem 1. If $f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n z^n \in \mathcal{MK}^{(k)}(A, B; \alpha; \beta)$, then

$$|a_1| \leq 1$$

and

$$(4) \quad |a_n| \leq \frac{2}{n(n+1)} + \frac{\beta(1-\alpha)(A-B)}{n} \left[1 + \sum_{k=1}^{n-1} \frac{2}{k+1} \right].$$

Proof. As $f \in \mathcal{MK}^{(k)}(A, B; \alpha; \beta)$, therefore (2) can be expressed as

$$\frac{-z^{2-k}f'(z)}{g_k(z)} = (P(z))^\beta,$$

which can be further represented as

$$(5) \quad \frac{-zf'(z)}{G_k(z)} = (P(z))^\beta,$$

where $G_k(z) = z^{k-1}g_k(z)$.

By Lemma 5, we have $G_k \in \mathcal{MS}^*$. For

$$(6) \quad q(z) = \frac{-zf'(z)}{G_k(z)},$$

we have

$$q(z) = 1 + \sum_{n=1}^{\infty} p_n z^n.$$

On expanding (6), it yields

$$(7) \quad \begin{aligned} \frac{1}{z} - a_1 z - 2a_2 z^2 - \cdots - na_n z^n - \cdots \\ = \left(\frac{1}{z} + b_1 z + b_2 z^2 + \cdots + b_n z^n + \cdots \right) \\ \times (1 + p_1 z + p_2 z^2 + \cdots + p_n z^n + p_{n+1} z^{n+1} + \cdots). \end{aligned}$$

As f is univalent in E^* , it is well known [3, 12] that $|a_1| \leq 1$.

Comparing the coefficients of z^n in (7), we have

$$(8) \quad -na_n = b_n + b_{n-1}p_1 + b_{n-2}p_2 + \cdots + b_2p_{n-2} + b_1p_{n-1} + p_{n+1}.$$

Applying triangle inequality and using Lemma 1 and Lemma 2 in (8), it gives

$$(9) \quad n|a_n| \leq \frac{2}{n+1} + \beta(1-\alpha)(A-B) \left[\frac{2}{n} + \frac{2}{n-1} + \cdots + \frac{2}{3} + 1 + 1 \right],$$

which proves Theorem 1. $\square$

For $A = (1 - 2\gamma)\eta$, $B = -\eta$, $\alpha = 0$, $\beta = 1$, Theorem 1 gives the following result:

Corollary 1. *If $f \in \mathcal{MK}^{(k)}(\eta, \gamma)$, then*

$$|a_1| \leq 1$$

and

$$|a_n| \leq \frac{2}{n(n+1)} + \frac{(2-2\gamma)\eta}{n} \left[1 + \sum_{k=1}^{n-1} \frac{2}{k+1} \right].$$

Putting $A = 1 - 2\gamma$, $B = -1$, $\alpha = 0$ and $\beta = 1$ in Theorem 1, the following result due to Yi et al. [14] is obvious:

Corollary 2. *If $f \in \mathcal{MK}^{(k)}(\gamma)$, then*

$$|a_1| \leq 1$$

and

$$|a_n| \leq \frac{2}{n(n+1)} + \frac{(2-2\gamma)}{n} \left[1 + \sum_{k=1}^{n-1} \frac{2}{k+1} \right].$$

Theorem 2. *If $-1 \leq B_2 = B_1 < A_1 \leq A_2 \leq 1$ and $0 \leq \alpha_2 \leq \alpha_1 < 1$, then*

$$\mathcal{MK}^{(k)}(A_1, B_1; \alpha_1; \beta) \subset \mathcal{MK}^{(k)}(A_2, B_2; \alpha_2; \beta).$$

Proof. As $f \in \mathcal{MK}^{(k)}(A_1, B_1; \alpha_1; \beta)$, so

$$\frac{-z^{2-k}f'(z)}{g_k(z)} \prec \left(\frac{1 + [B_1 + (A_1 - B_1)(1 - \alpha_1)]z}{1 + B_1z} \right)^\beta.$$

As $-1 \leq B_2 = B_1 < A_1 \leq A_2 \leq 1$ and $0 \leq \alpha_2 \leq \alpha_1 < 1$, we have

$$-1 \leq B_1 + (1 - \alpha_1)(A_1 - B_1) \leq B_2 + (1 - \alpha_2)(A_2 - B_2) \leq 1.$$

Thus by Lemma 3, it yields

$$\frac{-z^{2-k}f'(z)}{g_k(z)} \prec \left(\frac{1 + [B_2 + (A_2 - B_2)(1 - \alpha_2)]z}{1 + B_2z} \right)^\beta,$$

which implies $f \in \mathcal{MK}^{(k)}(A_2, B_2; \alpha_2; \beta)$. $\square$

Theorem 3. *If $f \in \mathcal{MK}^{(k)}(A, B; \alpha; \beta)$, then for $|z| = r, 0 < r < 1$, we have*

$$\begin{aligned} (10) \quad & \left(\frac{1 - [B + (A - B)(1 - \alpha)]r}{1 - Br} \right)^\beta \cdot \frac{(1 - r)^2}{r^2} \\ & \leq |f'(z)| \leq \left(\frac{1 + [B + (A - B)(1 - \alpha)]r}{1 + Br} \right)^\beta \cdot \frac{(1 + r)^2}{r^2} \end{aligned}$$

and

$$(11) \quad \int_0^r \left(\frac{1 - [B + (A - B)(1 - \alpha)]t}{1 - Bt} \right)^\beta \cdot \frac{(1 - t)^2}{t^2} dt$$

$$\leq |f(z)| \leq \int_0^r \left(\frac{1 + [B + (A - B)(1 - \alpha)]t}{1 + Bt} \right)^\beta \cdot \frac{(1 + t)^2}{t^2} dt.$$

Proof. From (5), we have

$$(12) \quad |f'(z)| = \frac{|G_k(z)|}{|z|} (P(z))^\beta.$$

Aouf [2] proved that

$$\frac{1 - [B + (A - B)(1 - \alpha)]r}{1 - Br} \leq |P(z)| \leq \frac{1 + [B + (A - B)(1 - \alpha)]r}{1 + Br},$$

which implies

$$(13) \quad \left(\frac{1 - [B + (A - B)(1 - \alpha)]r}{1 - Br} \right)^\beta \leq |P(z)|^\beta$$

$$\leq \left(\frac{1 + [B + (A - B)(1 - \alpha)]r}{1 + Br} \right)^\beta.$$

Since $G_k \in \mathcal{MS}^*$, so by Lemma 4, we have

$$(14) \quad \frac{(1 - r)^2}{r} \leq |G_k(z)| \leq \frac{(1 + r)^2}{r}.$$

(12) together with (13) and (14) yields (10). On integrating (10) from 0 to r , (11) follows. $\square$

For $A = (1 - 2\gamma)\eta$, $B = -\eta$, $\alpha = 0$, $\beta = 1$, Theorem 3 gives the following result due to Tang et al. [13].

Corollary 3. *If $f \in \mathcal{MK}^{(k)}(\eta, \gamma)$, then for $|z| = r$, $0 < r < 1$, we have*

$$\left(\frac{(1 - r)^2(1 - (1 - 2\gamma)\eta r)}{r^2(1 + \eta r)} \right) \leq |f'(z)| \leq \left(\frac{(1 + r)^2(1 + (1 - 2\gamma)\eta r)}{r^2(1 - \eta r)} \right)$$

and

$$\int_0^r \left(\frac{(1-t)^2(1-(1-(1-2\gamma)\eta t))}{t^2(1+\eta t)} \right) dt$$

$$\leq |f(z)| \leq \int_0^r \left(\frac{(1+t)^2(1+(1-(1-2\gamma)\eta t))}{t^2(1-\eta t)} \right) dt.$$

On putting $A = 1 - 2\gamma$, $B = -1$, $\alpha = 0$ and $\beta = 1$ in Theorem 3, the following result due to Yi et al. [14] is obvious:

Corollary 4. *If $f \in \mathcal{MK}^{(k)}(\gamma)$, then for $|z| = r, 0 < r < 1$, we have*

$$\frac{(1-r)^2[1-(1-2\gamma)r]}{r^2(1+r)} \leq |f'(z)| \leq \frac{(1+r)^2[1+(1-2\gamma)r]}{r^2(1-r)}$$

and

$$\int_0^r \frac{(1-t)^2[1-(1-2\gamma)t]}{t^2(1+t)} dt \leq |f(z)| \leq \int_0^r \frac{(1+t)^2[1+(1-2\gamma)t]}{t^2(1-t)} dt.$$

Theorem 4. *Let $f \in \mathcal{MK}^{(k)}(A, B; \alpha; \beta)$, then*

$$-Re \frac{(zf'(z))'}{f'(z)} \geq \begin{cases} -\frac{1+r}{1-r} - \beta \frac{(A-B)(1-\alpha)r}{(1-[B+(A-B)(1-\alpha)]r)(1-Br)}, & \text{if } R_1 \leq R_2, \\ -\frac{1+r}{1-r} + \frac{(A+B)-\alpha(A-B)}{(A-B)(1-\alpha)} \\ + 2 \frac{\sqrt{(1-B)(1-[B+(A-B)(1-\alpha)])(1+[B+(A-B)(1-\alpha)]r^2)(1+Br^2)}}{(A-B)(1-\alpha)(1-r^2)} \\ - 2 \frac{(1-[B+(A-B)(1-\alpha)]Br^2)}{(A-B)(1-\alpha)(1-r^2)}, & \text{if } R_1 \geq R_2, \end{cases}$$

where R_1 and R_2 are defined in Lemma 6.

Proof. As $f \in \mathcal{MK}^{(k)}(A, B; \alpha; \beta)$, we have

$$-zf'(z) = G_k(z)(P(z))^\beta.$$

Differentiating logarithmically, we get

$$(15) \quad -\frac{(zf'(z))'}{f'(z)} = \frac{zG'_k(z)}{G_k(z)} + \beta \frac{zP'(z)}{P(z)}.$$

As $G_k \in \mathcal{MS}^*$, we have

$$(16) \quad \operatorname{Re} \left(\frac{zG'_k(z)}{G_k(z)} \right) \geq -\frac{1+r}{1-r}.$$

Hence, using (16) and Lemma 6 in (15), the proof of Theorem 4 is obvious. $\square$

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