PREFACE

This volume grows out of the long-standing collaboration between the *Institute of Mathematical Sciences of the Americas (IMSA, University of Miami)* and the *International Center for Mathematical Sciences – Sofia (ICMS–Sofia)*, a partnership generously supported by the *Simons Foundation*. Over the past years this collaboration has fostered a series of initiatives designed not only to consolidate existing achievements, but also to open new directions for research in geometry and its neighboring fields. The conferences represented here may be seen as a natural continuation and deepening of this sustained effort.

The first conference, *Recent Developments in Hodge Theory*, was held in Sofia in August 9–11, 2023,
(`https://simons.icms.bg/event/recent-developments-in-hodge-theory/`).

The second, *Generalized and Symplectic Geometry*, took place in Sunny Beach in August 14–18, 2023
(`https://simons.icms.bg/event/generalized-and-symplectic-geometry/`).

Together, these meetings provided a forum for lively exchange across a wide spectrum of topics: from classical questions in topology and Hodge theory to emerging subjects such as tropical geometry and the developing theory of atoms.

The atmosphere was one of continuity and discovery. Long-standing problems in Hodge theory were revisited with new insight, while participants introduced techniques that are still in the process of taking shape. In particular, two themes crystallized during the conferences: the idea of a possible *non-Khler Hodge theory*, and the notion of *equivariant atoms*. These concepts remain at an early stage, yet they already point toward directions of research that will likely grow in significance.

A distinctive feature of these meetings was the balance between established authorities and younger voices, whose lectures together shaped the atmosphere of the conferences. In Sofia, Phillip Griffiths revisited central themes of Hodge theory, while Vestislav Apostolov and Gueo Grantcharov spoke on complex and symplectic geometry. Ernesto Lupercio introduced new directions linking Hodge-theoretic methods to non-Kähler contexts. The subsequent gathering in Sunny Beach broadened the scope still further: Ludmil Katzarkov and Tony Pantev addressed questions of homological mirror symmetry and stability conditions, Misha Shkolnikov presented developments in tropical geometry and sandpile models, and Morgan Brown offered contributions on o-minimal structures and derived equivalences. Taken together, these lectures dis-

played both continuity with classical traditions and a willingness to open new paths, a duality that is reflected in the contributions assembled in this volume.

The present volume gathers a selection of contributions inspired by the lectures and discussions of these meetings. Some reflect well-established areas of study; others record the first written appearances of ideas that surfaced in the course of the conferences themselves. Taken together, they offer not only a record of two memorable events, but also a glimpse of a field that is broadening its horizons and exploring new connections.

Tony Pantev
Ernesto Lupercio
Lino Grama